SEPARATION OF NON-NEWTONIAN LIQUIDS BY THERMAL DIFFUSION

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A transport equation has been derived for a mixture of non-Newtonian liquids showing power-law flow; the optimum separation conditions in a thermal-diffusion column in the presence of parasitic convection have been defined.

Thermal diffusion is used in separating and purifying isotropic and molecular mixtures, including oils, petroleum products, and solutions of polymers and other substances that constitute non-Newtonian liquids [1-3]. On the other hand, calculations on such columns and processing of results from such preparations are performed by means of the classical Jones–Furry theory [4], which does not incorporate the nonlinear viscosity. Here we consider the separation of liquids having Ostwald power-law rheology [5], which applies for a wide class of substances.

Consider the separation of a binary mixture of non-Newtonian liquids in a planar column of height L having a working gap of 2d with $d \ll L$, with the column closed at both ends. The left-hand wall is kept at temperature T_1 and the right-hand wall at T_2, with $T_1 < T_2$. The flow in the working gap is represented by negligible values of the transverse velocity component and temperature homogeneity in the longitudinal direction [6]. Then the following are the differential equations for the two-dimensional problem in (x, z) coordinates, which describe the quasistationary separation of these liquids in this thermal-diffusion column:

$$\frac{dP}{dz} = \frac{d}{dx}\, k\left(\frac{dv}{dx}\right)^n - \rho g\,[1 - \beta(T - T_0)], \qquad (1)$$

$$v\,\frac{\partial c}{\partial z} = \nabla D\left[\nabla c - \frac{\alpha c(1-c)}{T}\,\nabla T\right],$$

$$\frac{d^2T}{dx^2} = 0.$$

The parameters k, n, β, ρ, D, α in (1) are assumed to be constant and to be calculated for the average values of the temperature, pressure, and concentration.

The boundary conditions are:

$$v(d) = v(-d) = 0,$$

$$T(d) = \Theta, \quad T(-d) = -\Theta, \qquad (2)$$

$$\left[\frac{\partial c}{\partial x} - \frac{\alpha c(1-c)}{T}\,\frac{dT}{dx}\right]_{-d,\,d} = 0,$$

$$\int_{-d}^{d} v\,dx = 0, \qquad \int_{-d}^{d}\left(cv - D\,\frac{\partial c}{\partial z}\right)dx = 0.$$

The solution to the thermal-conduction equation is

$$T = T_0 + \Theta\,\frac{x}{d}\,. \qquad (3)$$

The dimensionless speed of the mixture $u = v(\rho g\beta\Theta d^{n+1}k^{-1})^{-1/n}$ is given by the following equation [7]:

$$u = \int_0^{\eta}\left|A - \frac{\eta^2}{2}\right|^{1/n}\operatorname{sign}\left(A - \frac{\eta^2}{2}\right)d\eta, \qquad (4)$$

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TABLE 1. Values for the Constant of Integration A, Dimensionless Mean Speed $\bar{u}$, and Integrals $\varphi(n)$ and $\psi(n)$

n	A	$\bar{u}$	$\varphi(n)$	$\psi(n)$
0,5	0,1862	$8{,}106\cdot10^{-3}$	$4{,}393\cdot10^{-3}$	$2{,}683\cdot10^{-5}$
0,6	0,1810	$1{,}386\cdot10^{-2}$	$7{,}491\cdot10^{-3}$	$7{,}851\cdot10^{-5}$
0,7	0,1766	$2{,}045\cdot10^{-2}$	$1{,}101\cdot10^{-2}$	$1{,}706\cdot10^{-4}$
0,8	0,1728	$2{,}746\cdot10^{-2}$	$1{,}474\cdot10^{-2}$	$3{,}073\cdot10^{-4}$
0,9	0,1695	$3{,}458\cdot10^{-2}$	$1{,}851\cdot10^{-2}$	$4{,}866\cdot10^{-4}$
1,0	0,1667	$4{,}167\cdot10^{-2}$	$2{,}222\cdot10^{-2}$	$7{,}052\cdot10^{-4}$
1,1	0,1641	$4{,}858\cdot10^{-2}$	$2{,}584\cdot10^{-2}$	$9{,}565\cdot10^{-4}$
1,2	0,1620	$5{,}528\cdot10^{-2}$	$2{,}933\cdot10^{-2}$	$1{,}237\cdot10^{-3}$
1,3	0,1600	$6{,}171\cdot10^{-2}$	$3{,}267\cdot10^{-2}$	$1{,}538\cdot10^{-3}$
1,4	0,1582	$6{,}786\cdot10^{-2}$	$3{,}585\cdot10^{-2}$	$1{,}857\cdot10^{-3}$
1,5	0,1566	$7{,}363\cdot10^{-2}$	$3{,}879\cdot10^{-2}$	$2{,}182\cdot10^{-3}$
1,6	0,1551	$7{,}919\cdot10^{-2}$	$4{,}161\cdot10^{-2}$	$2{,}520\cdot10^{-3}$
1,7	0,1538	$8{,}446\cdot10^{-2}$	$4{,}426\cdot10^{-2}$	$2{,}861\cdot10^{-3}$

where $\eta = x/d$; Table 1 gives the values calculated for A on the basis that the liquid adheres to the wall, as well as the dimensionless mean speed $\bar{u}=\int_0^1 u d\eta$ as a function of the index n. The mean speed increases with n, but the shape of the velocity profile alters, while the flow is antisymmetrical about $\eta = 0$.

Integration of (1) subject to (2) by the method of [6] gives the transport equation as

$$\tau = Hc(1-c)-(K_c+K_d)\frac{dc}{dz}, \tag{5}$$

where

$$H=\frac{2\,\alpha\rho\Theta Bd}{T_0}\left(\frac{\rho g\beta\Theta d^{n+1}}{k}\right)^{1/n}\varphi(n), \quad \varphi(n)=-\int_0^1 d\eta\int_{-1}^{\eta} ud\eta,$$

$$K_c=\frac{2\rho Bd^3}{D}\left(\frac{\rho g\beta\Theta d^{n+1}}{k}\right)^{2/n}\psi(n), \quad \psi(n)=\int_0^1 d\eta\left[\int_{-1}^{\eta} ud\eta\right]^2,$$

$$K_d=2\rho DBd.$$

Table 1 gives the values of the integrals $\varphi(n)$ and $\psi(n)$ computed for various values of n.

If (5) is integrated with respect to z for the steady state ($\tau = 0$), we get an expression for the separation factor q^* for these non-Newtonian liquids:

$$Y=\ln q^*=\frac{HL}{K_c+K_d}. \tag{6}$$

The separation factor is substantially dependent on the dimensions of the column; the maximum value for q^* occurs for a certain optimum half-width d_0 for the gap, and the condition for the occurrence of this is

$$K_d=\frac{3n+1}{n+1}K_c. \tag{7}$$

Condition (7) becomes $K_d = 2K_c$ for Newtonian liquids [4]. We combine (5) and (7) to get an expression for the optimum width:

$$d_0=\left[\frac{n+1}{3n+1}\,\frac{D^2}{(\rho g\beta\Theta k^{-1})^{2/n}\psi(n)}\right]^{\frac{n}{4n+2}}. \tag{8}$$

These equations relate to an ideal column, whereas an actual system inevitably has parasitic mixing on account of variations in the working gap over the length of the column and uneven heating in the azimuthal direction, etc. The separation factor in the presence of parasitic convection becomes [6]

$$q=\frac{1+\varkappa}{1-\varkappa}\,\frac{1-2\varkappa+\exp[(1-\varkappa)Y]}{1+2\varkappa+\exp[-(1+\varkappa)Y]}, \tag{9}$$

where $\varkappa$ characterizes the effect of the parasitic convection and is inversely related to the separation factor. The expression for $\varkappa$ for non-Newtonian liquids is

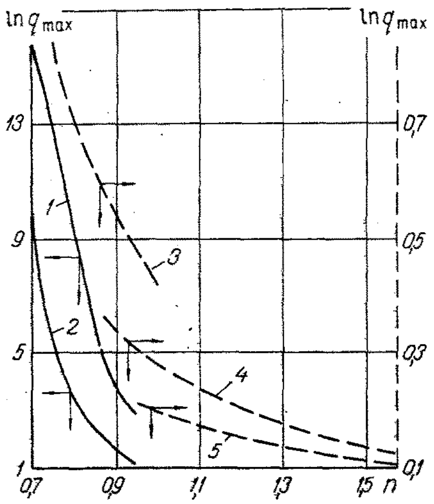

Fig. 1. Logarithm of the separation factor at the optimum gap width as a function of n for the following values of k: 1) $k = 10^{-1}$; 2) $3 \cdot 10^{-2}$; 3) $3 \cdot 10^{-3}$; 4) 10^{-3}; 5) $3 \cdot 10^{-4}$.

$$\varkappa = \frac{\bar{u}}{\alpha\varphi(n)}\frac{T_0}{\Theta}\left(\frac{2\delta T}{\Theta}\right)^{1/n}. \tag{10}$$

As $\delta T \ll \Theta$, while $\bar{u}(n)$ is proportional to $\varphi(n)$, we have that n and $\varkappa$ increase together. Therefore, if the liquids are pseudoplastic ($n < 1$), the effects of the parasitic convection are less important than are those for dilatant liquids ($n > 1$), with Newtonian liquids occupying an intermediate position. In the case of Newtonian liquids ($n = 1$, $\bar{u} = 1/24$, $\varphi = 1/45$) (10) becomes a standard relation [6]:

$$\varkappa = 15\frac{T_0\delta T}{\alpha(2\Theta)^2}.$$

It is best to operate a thermal-diffusion column with convective heating of the hot wall [6], and in that case the temperature variation in the azimuthal direction (temperature difference δT) is [8] given by

$$\delta T = \frac{\varepsilon\Theta}{d(1+2ds)}, \tag{11}$$

where

$$s = \frac{h_1 h_2}{\lambda(h_1+h_2)}; \quad h_j = \frac{\alpha_j}{1+\mathrm{Bi}_j}.$$

Equations (5), (6), (9), (10), and (11) define the conditions for obtaining the maximum separation factor for such liquids in the presence of parasitic convection; the dependence of q on d, n, α, and L was examined numerically by computer. The consistency parameter k characterizes the set of values typical for the restricted range in n employed here; in particular, k was taken as constant for the various n, although the dimensions of the consistency parameter in the SI system are dependent on n. The other parameters were kept constant and took the following characteristic values: $D = 10^{-9}$ m^2/sec; $\beta = 10^{-3}$ deg^{-1}; $\rho = 10^3$ kg/m^3; $\Theta = 50$°K; $T_0 = 350$°K, $\varepsilon = 10^{-5}$ m; $s = 2 \cdot 10^{-4}$ m^{-1}. The illustrations present the results.

Figure 1 shows ln q_{max} for the optimum gap width in relation to the flow index for various values of k and $\alpha = 0.1$, $L = 1$ m. Figure 2 shows the optimum half-width as a function of n for $\alpha = 0.1$ and $L = 1$ m.

These results show that for small values of the consistency parameter ($k < 0.3$) the coefficients in the transport equation are subject to the inequality $K_d \ll K_c$; in that case, the mean flow speed $\bar{v}$ increases with n, which results in a fall in the separation factor Y for an ideal column. On the other hand, $\varkappa$ tends to increase with n, and the overall result is an increase in q as n decreases. In the present case, the optimum gap width is dependent on H, K_c, and $\varkappa$, and in particular d_0 increases with the flow index.

At higher values of the consistency parameter ($k > 0.3$), the increase in the frictional force tends to depress the circulation in the column, and in that case the maximum separation is attained on increasing the working gap, and further the optimum half-width increases as n decreases, with the limit set by the value given by (8). Therefore, separation of pseudo-

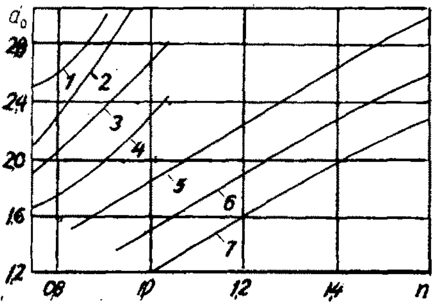

Fig. 2

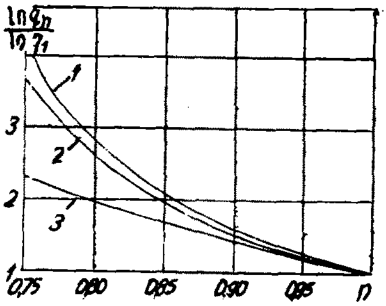

Fig. 3

Fig. 2. Optimum gap half-width as a function of n for the following values of k: 1) 10^{-1}; 2) $3 \cdot 10^{-2}$; 3) 10^{-2}; 4) $3 \cdot 10^{-3}$; 5) 10^{-3}; 6) $3 \cdot 10^{-4}$; 7) 10^{-4}; d_0, 10^{-4} m.

Fig. 3. Ratio of the logarithms of the separation factors for pseudoplastic and Newtonian liquids as a function of n for $2d = 3 \cdot 10^{-4}$ m for the following values of k: 1) 10^{-2}; 2) $3 \cdot 10^{-3}$; 3) 10^{-3}.

plastic liquids having a high value for the consistency parameter is possible in a thermal diffusion column with little effect from the parasitic mixing: d is governed by H, K_c, and K_d. The maximum separation factor as a function of n itself has a pronounced peak, which is defined by (7), and this peak shifts to higher values of n as the consistency parameter increases.

The numerical study of the effects from thermal-diffusion constant α and column height L showed that the parasitic mixing has less effect as this constant increases and as L is reduced, which has been demonstrated previously [6] for Newtonian liquids.

As a rule, molecular mixtures are usually separated by thermal diffusion in columns in which the working gap is $2d = 3 \cdot 10^{-4}$ m, i.e., not the optimal value; Fig. 3 shows the ratio of the logarithms of the separation factors for pseudoplastic and Newtonian liquids $\ln q_n / \ln q_1$ as a function of n as calculated for a column with $2d = 3 \cdot 10^{-4}$ m and various values of the consistency parameter. Clearly, pseudoplastic liquids are more readily separated by thermal diffusion than are Newtonian ones under otherwise equal conditions.

NOTATION

v, longitudinal velocity component; P, pressure; T, absolute temperature; $T_0 = (T_1 + T_2)/2$, mean temperature of mixture; c, concentration; g, gravitational acceleration; $\beta = -(1/\rho)(\partial\rho/\partial T)$, thermal expansion coefficient; k, consistency index; n, flow index; D, diffusion coefficient; α, thermal-diffusion factor; $\Theta = (T_2 - T_1)/2$; u, dimensionless velocity; $\bar{u}$, dimensionless mean velocity; x, horizontal coordinate; z, vertical coordinate; τ, longitudinal flow of lighter component; H, K_c, and K_d, coefficients in (5); q^*, separation coefficient for an ideal column; q, separation coefficient for a column with parasitic convection; $Y = \ln q^*$; ϰ, parasitic-convection parameter; δT, azimuthal temperature difference; B, slot width; ε, mean deviation from normal gap width; λ, thermal conductivity of mixture; α_j, heat-transfer coefficients for thermostatic surfaces (j = 1, 2); $Bi_j = \alpha_j\delta_j/\lambda_j$, δ_j; λ_j, wall thickness and thermal conductivities.

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